Week 10:

Faraday's law of induction

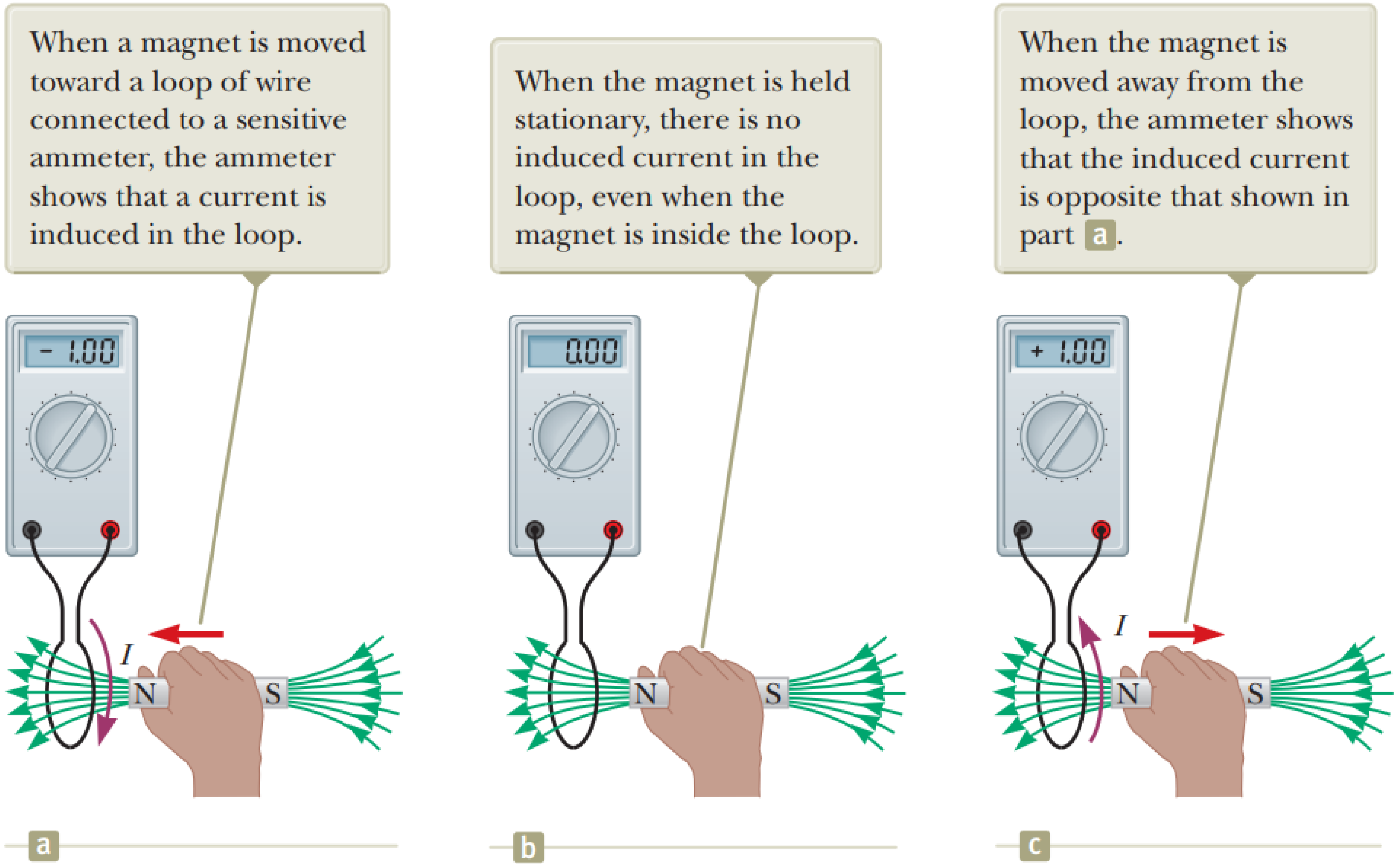
Faraday's Law of Induction

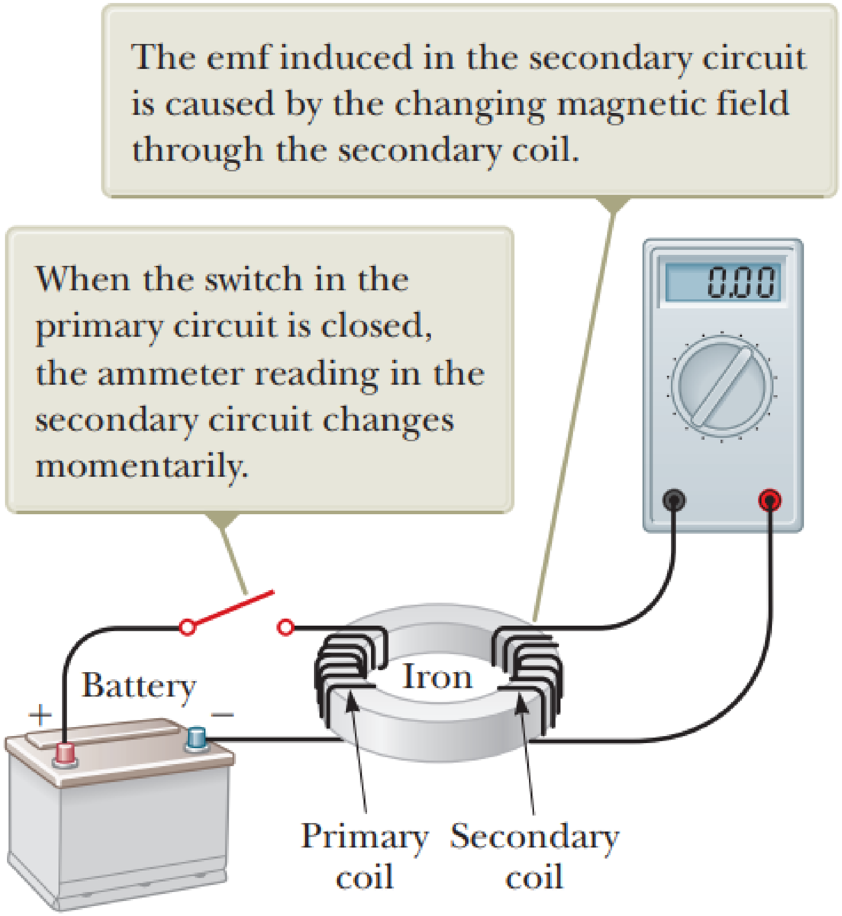
DEMO

https://auditoires-physique.epfl.ch/experiment/612/induction-terrestre

https://auditoires-physique.epfl.ch/experiment/226/mise-en-evidence-de-linduction-ii

https://auditoires-physique.epfl.ch/experiment/568/mise-en-evidence-de-linduction-i



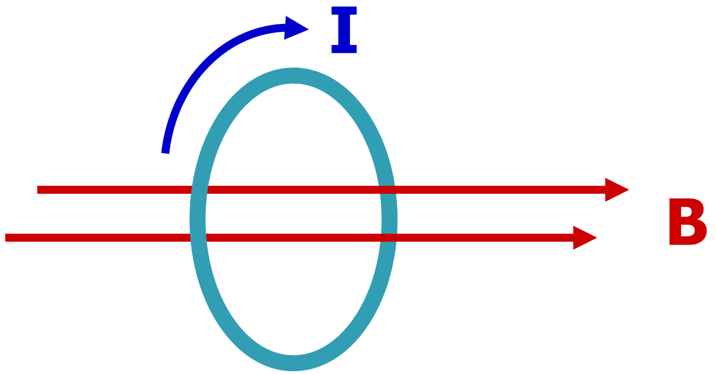


Magnetic Induction

- in the previous lectures, we found that electric current can give rise to a magnetic field

Question: Can magnetic field give rise to an electric current?

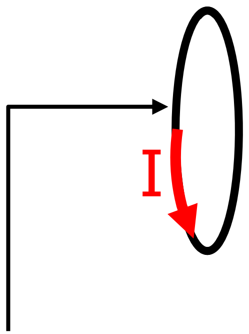
Observation: electric current is induced if magnetic flux through closed circuit (e.g., loop of wire) **changes**



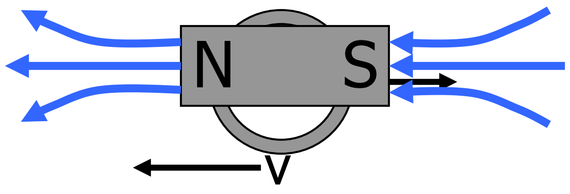
- **constant** magnetic flux does **not** induce a current

"Change" may or may not require observable (macroscopic) motion

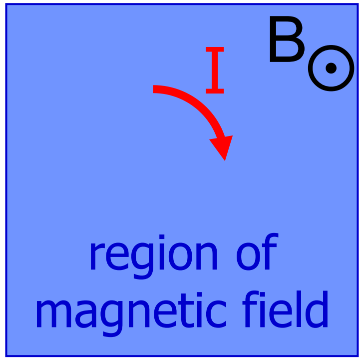
- a magnet may move through a loop of wire
- a loop of wire may be moved through a magnetic field



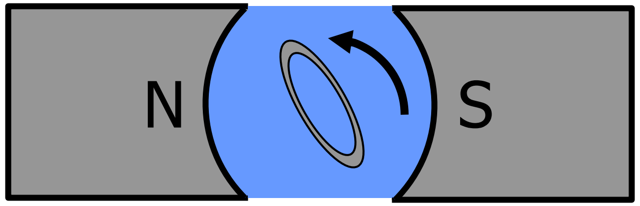
this part of the loop is closest to your eyes



move magnet toward coil

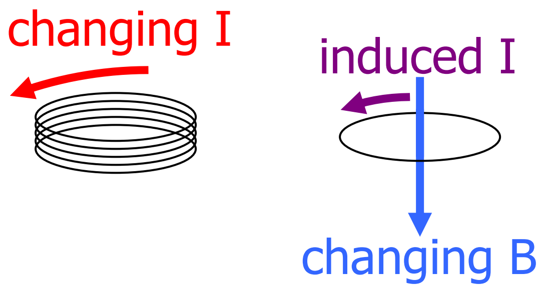


change area of loop inside magnetic field



rotate coil in magnetic field

Magnetic Induction



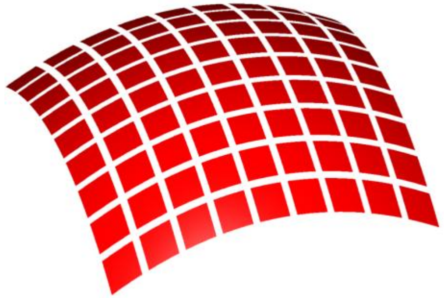
- changing current in wire loop gives rise to changing magnetic field which can induce a current in another nearby loop of wire

In this case, nothing observable (to your eye) is moving, although of course microscopically, electrons are in motion.

Induced current is produced by a changing magnetic flux.

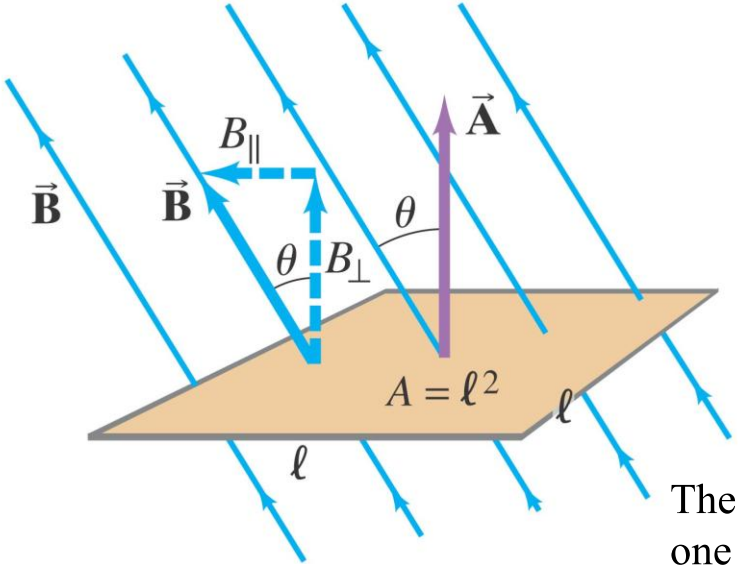
Magnetic flux (open surface)

Magnetic Flux through a loop (open surface): $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$



Unit of magnetic flux: Weber, Wb: 1 Wb = 1 T·m^2.

The variables in the flux definition:



Determining the flux through a flat loop of wire. This loop is square, of side ℓ and area $A = \ell^2$.

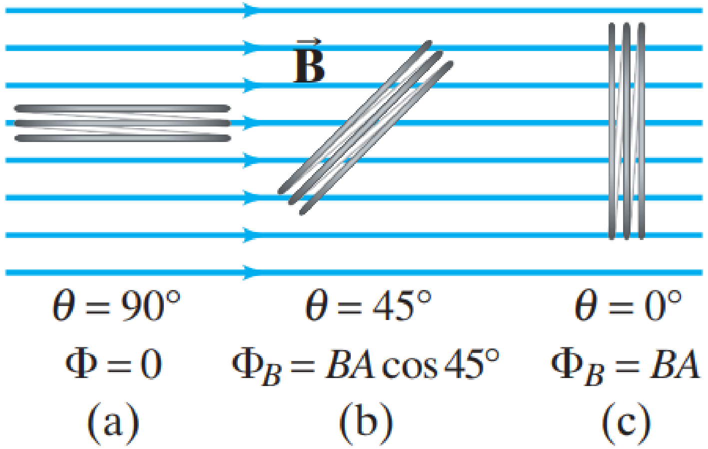
- Vector $\vec{A}$ is in the direction of $\vec{B}$

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA\cos\theta$$

always positive

The integral is taken **over an open surface**—that is, one **bounded by a closed curve** such as a circle or square.
In the present discussion, the area is that enclosed by the loop under discussion.
The area is not an enclosed surface as we use in Gauss's law.

Magnetic flux Φ_B is proportional to the number of lines of $\vec{\mathbf{B}}$ that pass through the loop.



Gauss's Law in Magnetism (closed surface)

The magnetic field lines generated by a current and of a bar magnet do not begin or end at any point. **For any closed surface, the number of lines entering the surface equals the number leaving the surface**; therefore, the **net magnetic flux is zero**. In contrast, for a closed surface surrounding one charge of an electric, the net electric flux is not zero.

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

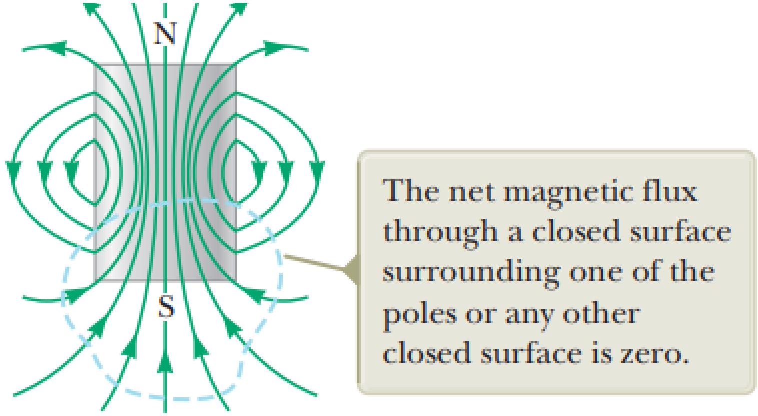


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

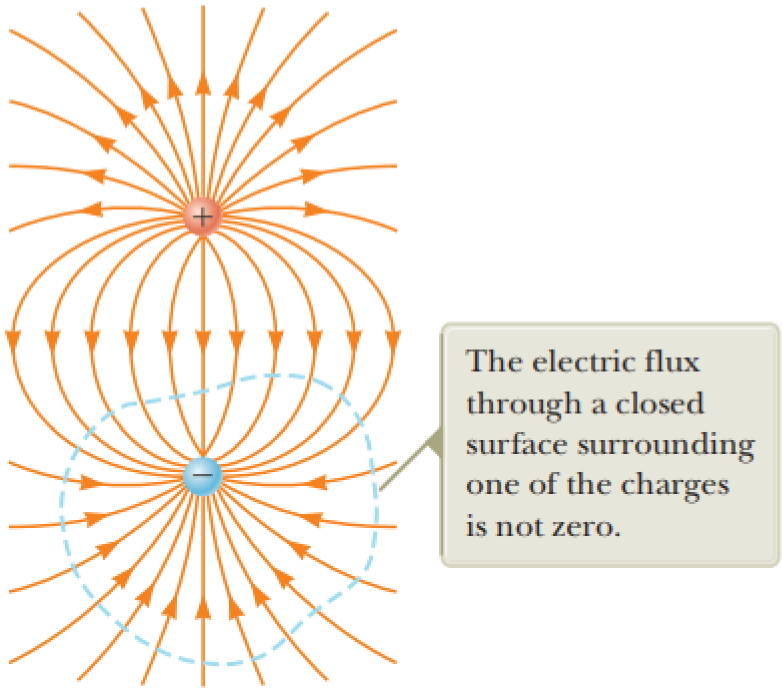
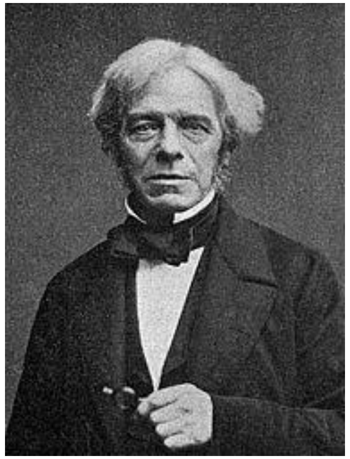


Figure 30.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

Faraday's Law of Induction

Faraday's law of induction: the *emf* $\mathcal{E}$ induced in a circuit is equal to the rate of change of magnetic flux through the circuit:

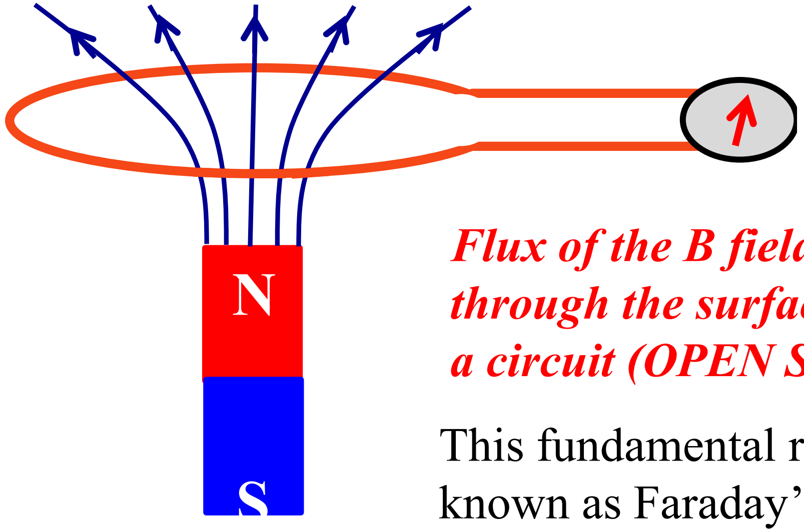


$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

or

If the circuit contains N *loops* that are closely wrapped so the same flux passes through each, the ***emfs*** induced in each loop add together, so

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \quad [N \text{ loops}]$$



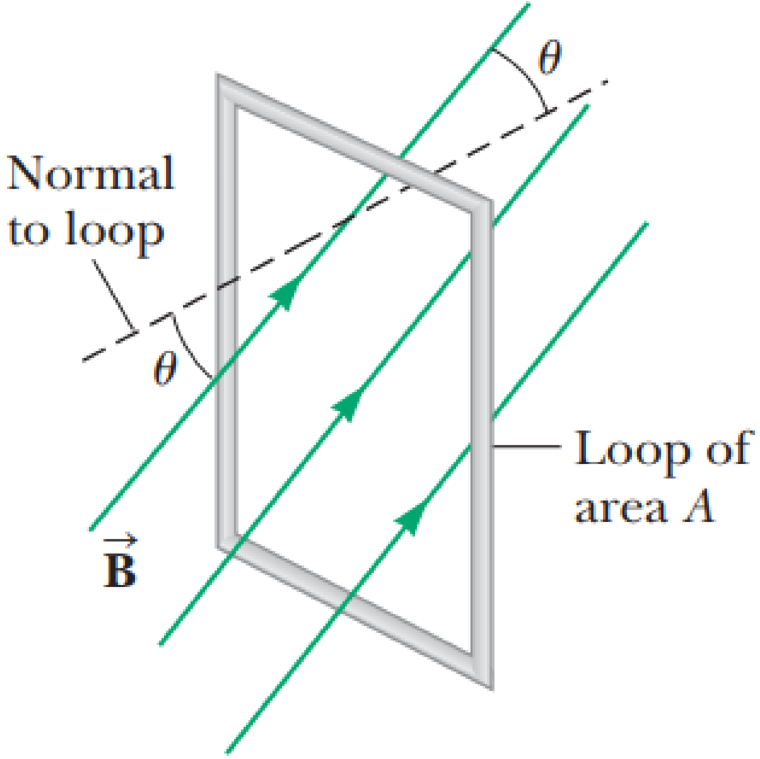
Flux of the B field calculated through the surface bounded by a circuit (OPEN SURFACE)

This fundamental result is known as Faraday's law of induction and is one of the basic laws of electromagnetism.

emf $\mathcal{E}$ ***= electromotive force***

The phrase ***electromotive force*** is an unfortunate historical term, describing not a force, but rather ***a potential difference in volts***.

Faraday's Law of Induction



Suppose a loop enclosing an area A lies in a uniform magnetic field **B** as in Figure.
The magnetic flux through the loop is equal to $BAcos\theta$, where θ is the angle between the magnetic field and the normal to the loop;
hence, the induced *emf* $\mathcal{E}$ can be expressed as

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

From this expression, we see that an *emf* can be induced in the circuit in several ways:

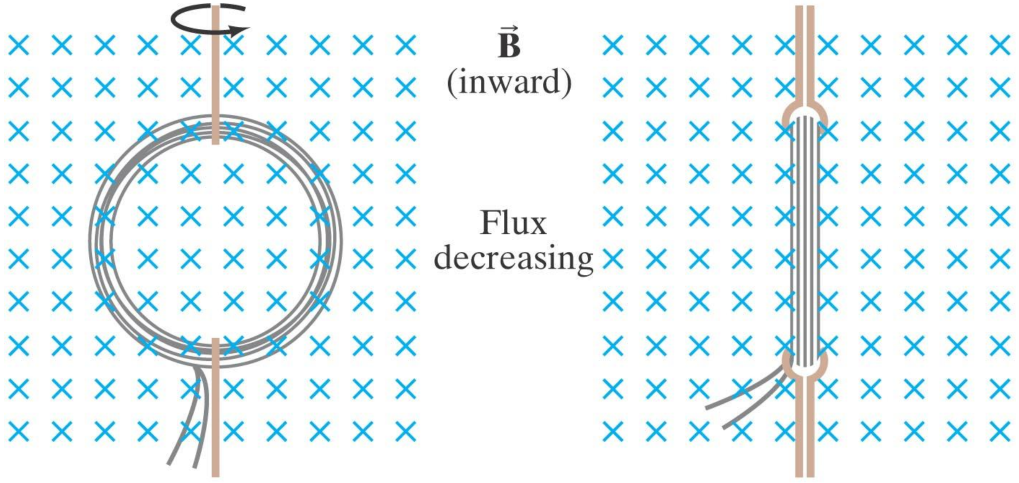
- The magnitude of **B** can change with time.
- The area A enclosed by the loop can change with time.
- The angle θ between **B** and the normal to the loop can change with time.
- Any combination of the above can occur.

Faraday's Law of Induction

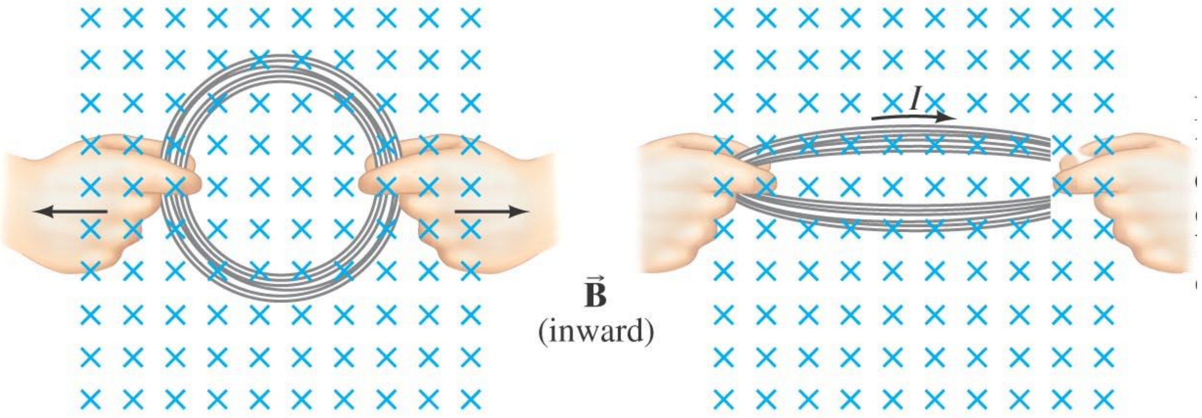
Magnetic flux will change if the **angle** between the loop and the field changes.

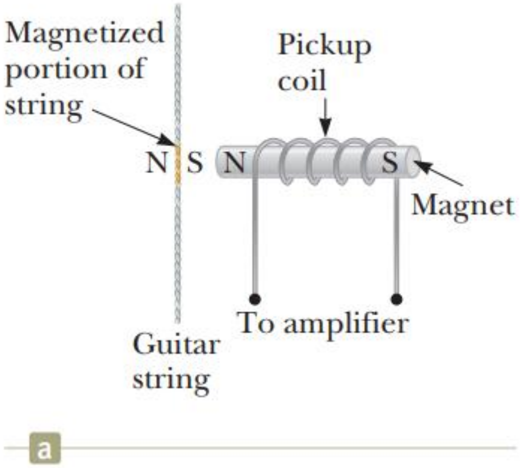
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnetic flux will change if the **area** of the loop changes.



(a) Maximum flux (b) Zero flux





Magnetic flux will change if the **intensity of the magnetic field** through the pickup loop changes.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}}{dt}$$

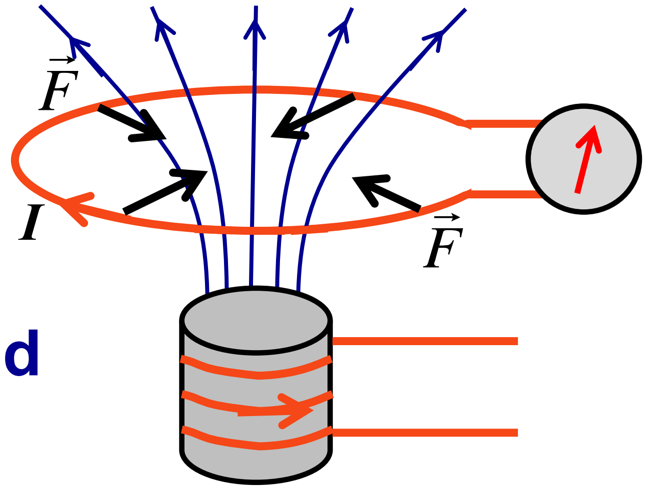
Lenz's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

The direction of the induced reaction (i.e. current) is such as to oppose the change in flux through the loop.

How the system compensate for it:

1. **Generate its own opposite magnetic field**
2. **Make the loop smaller !**



$$I = \frac{emf}{R} = \frac{1}{R}\frac{d}{dt}\int B \cdot dA = \frac{1}{R}\int B\,\frac{dA}{dt}$$

Solenoid is switching on:
Flux increases

Demo https://auditoires-physique.epfl.ch/experiment/608/tour-de-jufer-courants-induits

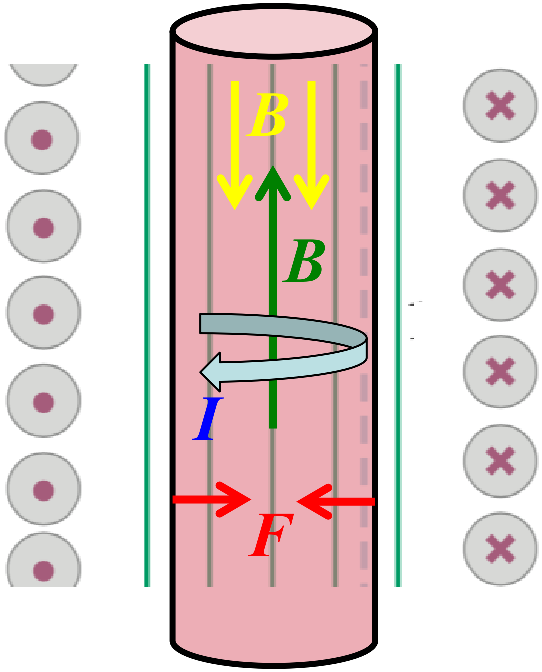
Aluminum Can in a Magnetic Field

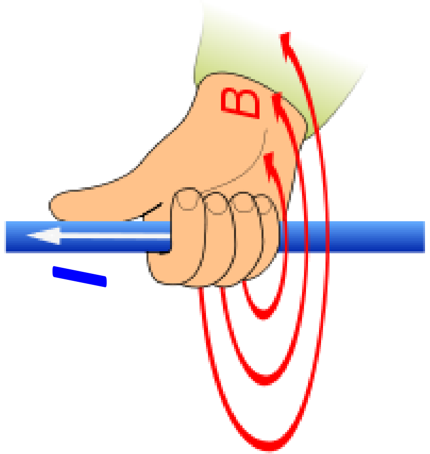
Current *i* changes from 0 to i_f

Demo

https://auditoires-physique.epfl.ch/experiment/355/forces-de-lorentz-sur-canette

- Current *i* in solenoid makes magnetic field (Ampere-Law)
- Change of magnetic field induces *emf* (Faraday) and a current *I=emf* /R (Ohm) in the coke-can
- Direction of *I* is opposite to *i* (Lenz); Induced current makes induced magnetic field (Ampere) to reduce the solenoid magnetic field inside the can.
- Due to the induced current in the can, there is a magnetic force on this current is toward inside of the coke-can (Lorentz)

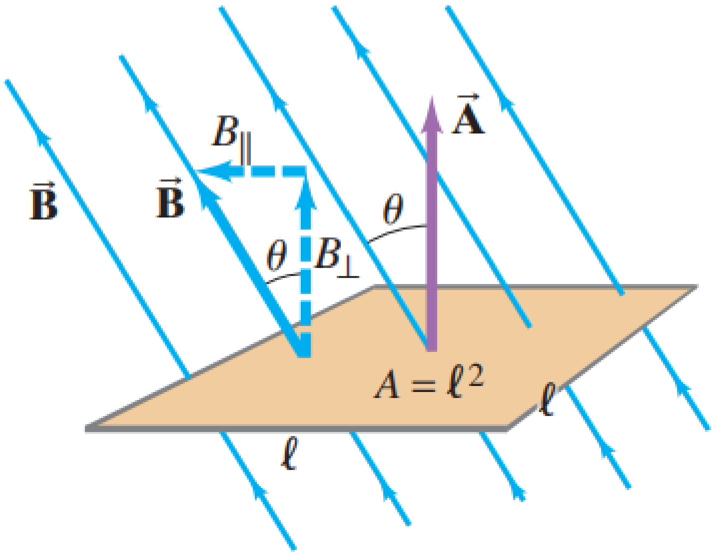




Coke-can resists to the increase of the magnetic flux by reducing its cross-section !

A loop of wire in a magnetic field

A square loop of wire of side *l=5.0 cm* is in a uniform magnetic field *B=0.16T*. What is the magnetic flux in the loop (a) when ***B*** is perpendicular to the face of the loop and (b) when ***B*** is at an angle of 30° to the area ***A*** of the loop?
(c) What is the magnitude of the average current in the loop if it has a resistance of 0.012 Ω and it is rotated from position (b) to position (a) in 0.14 s?



SOLUTION The area of the coil is $A = \ell^2 = (5.0 \times 10^{-2}\,\text{m})^2 = 2.5 \times 10^{-3}\,\text{m}^2$, and the direction of $\vec{\mathbf{A}}$ is perpendicular to the face of the loop (Fig.).
(*a*) $\vec{\mathbf{B}}$ is perpendicular to the coil's face, and thus parallel to $\vec{\mathbf{A}}$ (Fig.), so

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$
$$= BA\cos 0° = (0.16\,\text{T})(2.5 \times 10^{-3}\,\text{m}^2)(1) = 4.0 \times 10^{-4}\,\text{Wb}.$$

(*b*) The angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ is 30°, so

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$
$$= BA\cos\theta = (0.16\,\text{T})(2.5 \times 10^{-3}\,\text{m}^2)\cos 30° = 3.5 \times 10^{-4}\,\text{Wb}.$$

(*c*) The magnitude of the induced emf is

$$\mathscr{E} = \frac{\Delta\Phi_B}{\Delta t} = \frac{(4.0 \times 10^{-4}\,\text{Wb}) - (3.5 \times 10^{-4}\,\text{Wb})}{0.14\,\text{s}} = 3.6 \times 10^{-4}\,\text{V}.$$

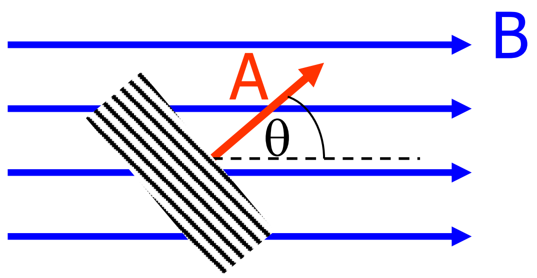
The current is then

$$I = \frac{\mathscr{E}}{R} = \frac{3.6 \times 10^{-4}\,\text{V}}{0.012\,\Omega} = 0.030\,\text{A} = 30\,\text{mA}.$$

emf Generators

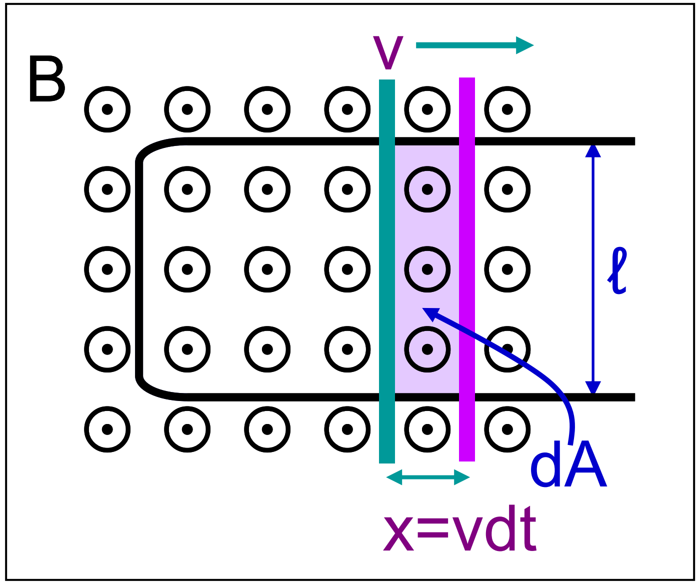
- a generator is a device that produces electricity (specifically, an emf) from motion

1. Flux change if a wire loop rotates in a magnetic field



Rotating loop generator

2. Flux change if a wire loop expands or contracts in a magnetic field

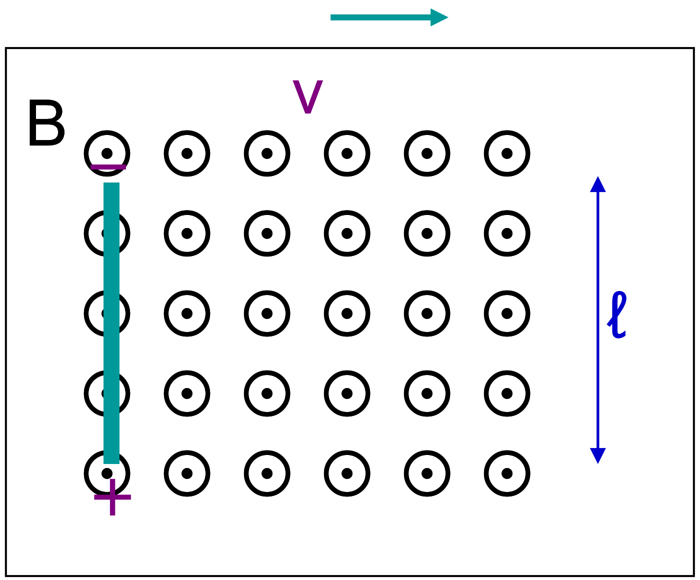


slide-wire generator

emf Generators

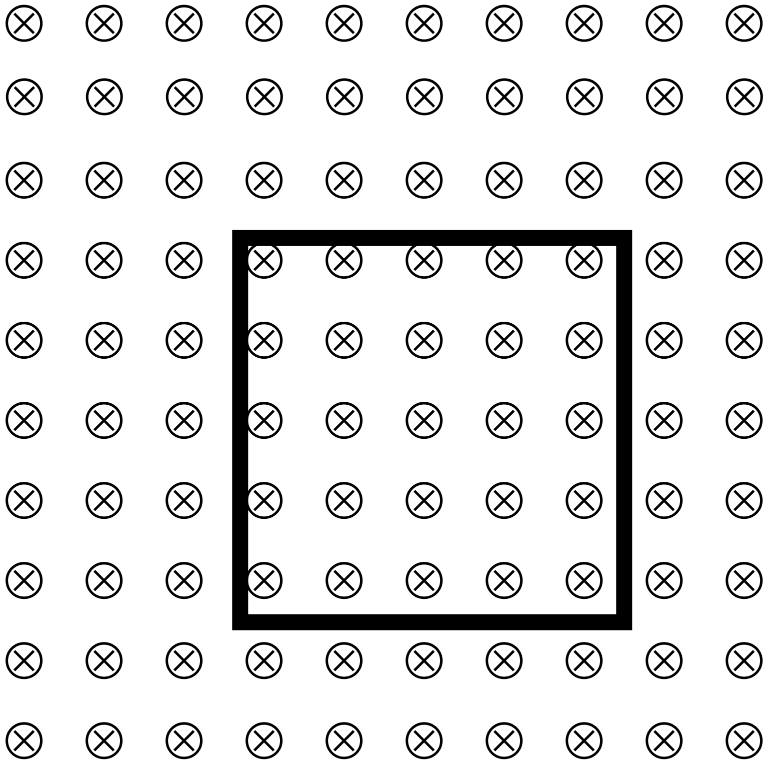
- a generator is a device that produces electricity (specifically, an emf) from motion

3. Conductor moving in a magnetic field: motional emf

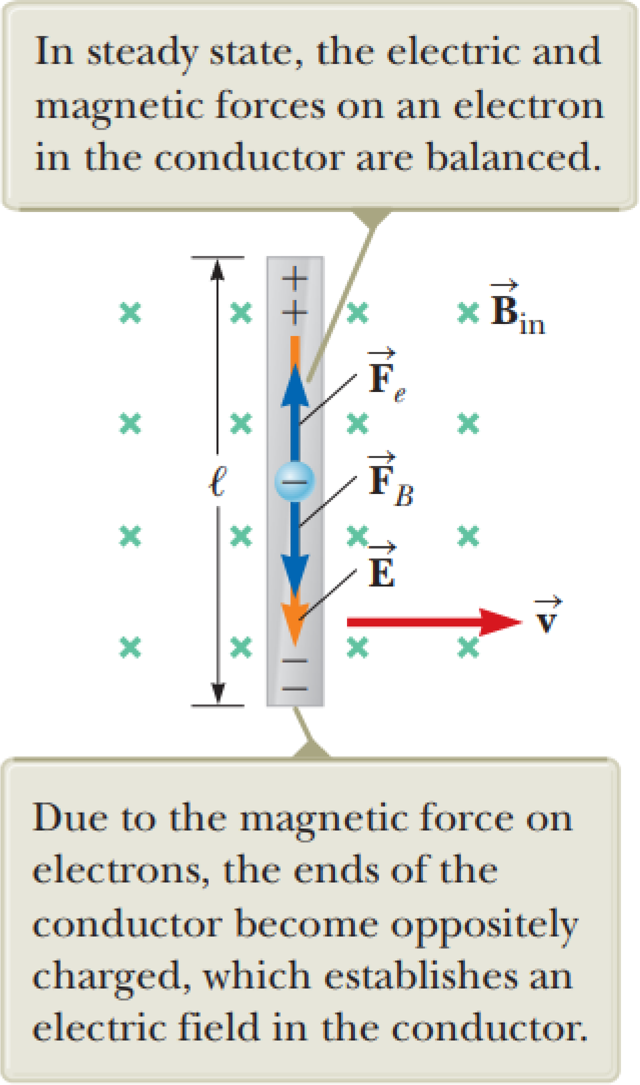


motion produces voltage difference between wire ends

4. Flux change if a wire loop moves into or out of a magnetic field



Motional *emf*



An ***emf*** can be induced in a conductor moving through a constant magnetic field

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end.
- As a result of this charge separation, an electric field ***E*** is produced inside the conductor.

$$qE = qvB \quad \text{or} \quad E = vB$$

$$\Delta V = E\ell = B\ell v$$

- A potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.
- If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

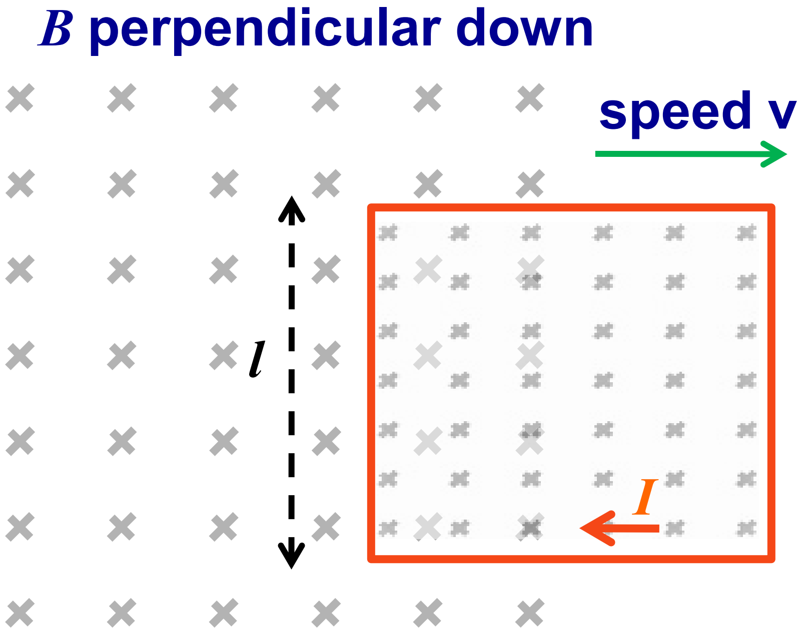
Pulling a coil out from a Magnetic Field

- A square loop of side l is moving at speed v <u>out</u> of a region of uniform field $\boldsymbol{B}$.
- The induced emf is

$$\mathcal{E} = -d\Phi_B / dt = Bvl$$

- The direction of current is such to induce a magnetic field in the same direction of the external one

(to compensate for the variation-- Lenz's law).



In time dt the loop will move distance vdt, so the area outside of magnetic field will be $vdt \cdot l$.

Example: *emf* Induced in a Moving Conductor

Force on moving rod in magnetic field.

(a) Determine the magnitude of the required force.

(b) What external power is needed to move the rod?

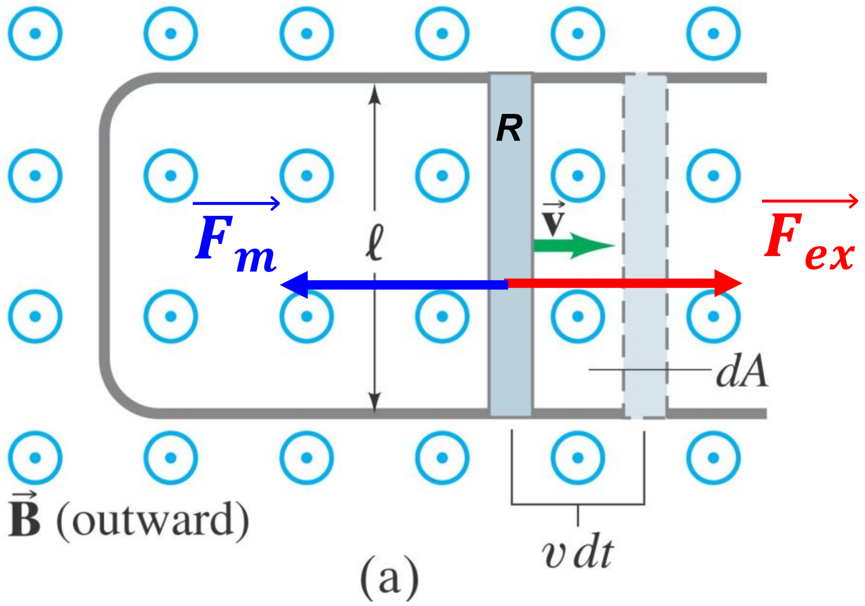
$$\overrightarrow{F_m} = I\vec{l} \times \vec{B}$$

(a) $\Phi = \vec{B}\cdot\vec{S}, \quad \varepsilon = -\frac{d\Phi}{dt}, \quad I = \frac{\varepsilon}{R}, \quad F_m = IBl\sin\theta$

$$\frac{d\Phi}{dt} = B\frac{dA}{dx}\frac{dx}{dt} = B\frac{d(l\cdot x)}{dx}v = B\cdot l\cdot v$$

$$I = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{-Blv}{R}$$

$$F_{ex} = -F_{el} = BlI = \frac{\underline{BlBlv}}{R} = \frac{B^2l^2v}{R}$$



(b) $P = F\cdot v = I^2R$

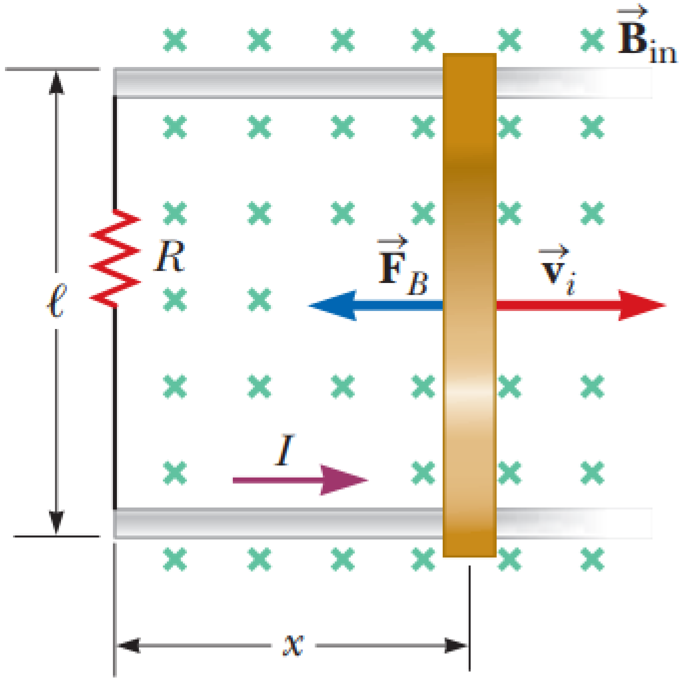
$E_{kin} = 1/2\cdot mv^2$; $\Sigma F = 0 \Rightarrow v = \text{const.}$

EPFL

Example: Magnetic Force Acting on a Sliding Bar

The conducting bar illustrated in Figure moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m, and its length is l. The bar is given an initial velocity $\boldsymbol{v}$ to the right and is released at t = 0.

The magnetic force is $F_B = -IlB$, where the negative sign indicates that the force is to the left. The magnetic force is the only horizontal force acting on the bar.



Using the particle under a net force model, apply Newton's second law to the bar in the horizontal direction:

$$F_x = ma \quad \rightarrow \quad -I\ell B = m\frac{dv}{dt}$$

Substitute $I = B\ell v/R$ from previous example

$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$

Rearrange the equation so that all occurrences of the variable v are on the left and those of t are on the right:

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

Integrate this equation using the initial condition that $v = v_i$ at $t = 0$ and noting that $(B^2\ell^2/mR)$ is a constant:

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

Define the constant $\tau = mR/B^2\ell^2$ and solve for the velocity:

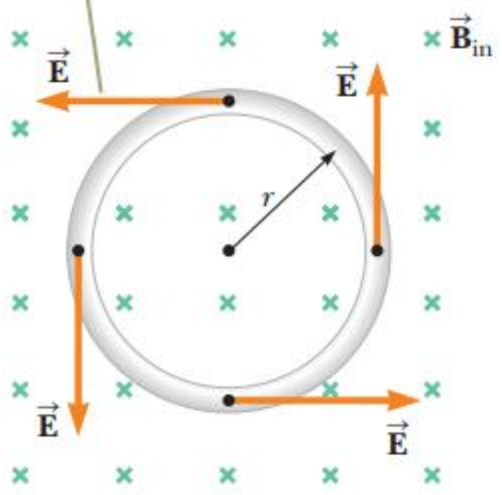
$$(1) \quad v = v_i e^{-t/\tau}$$

This expression for $\boldsymbol{v}$ indicates that the velocity of the bar decreases with time under the action of the magnetic force

Relation between Electric and Magnetic fields

- **A Change of Magnetic Flux Produces an Electric Field**

The induction of a current in the loop implies the presence of an induced electric field $\vec{E}$, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force.



Kirchhoff's rule:

The emf for any closed path can be expressed as the line integral of $\vec{E} \cdot \vec{dl}$ over that path

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\ell}$$

Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

This is a generalization of Faraday's law.

The electric field will exist regardless of whether there are any conductors around.

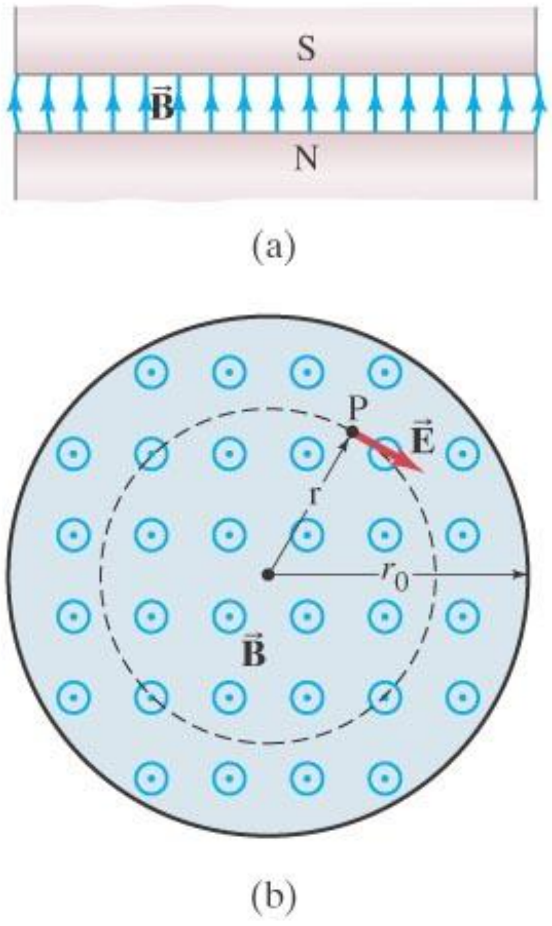
The induced electric field $\vec{E}$ is a **nonconservative** field that is generated by a changing magnetic field. The field $\vec{E}$ that satisfies the generalization of Faraday's law cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral over a closed loop would be zero.

Example: *E* field produced by changing *B* field

Magnetic field between two round poles of an electromagnet is nearly uniform.

The current in the coil of the electromagnet alternates as: $I = I_0 \cos\omega t$. This generate a magnetic field B(t) ∝ *I*(t). Maximum field is B_0.

Determine the electric field at any point P at distance *r* from the center of the poles.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}; \quad \oint \vec{\mathbf{E}} \cdot d\vec{\ell} = 2\pi r E$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}\pi r^2$$

$$B = \alpha I = \alpha I_0 \cos\omega t$$

$$B_0 = \alpha I_0 \Longrightarrow \boxed{B = B_0 \cos\omega t}; \quad \frac{dB}{dt} = -\omega B_0 \sin\omega t$$

$$\omega B_0 \pi r^2 \sin\omega t = 2\pi r E \Longrightarrow \boxed{E = \frac{1}{2} r\omega B_0 \sin\omega t}$$

Example: Motional emf Induced in a Rotating Bar

A conducting bar of length l, rotates with a constant angular speed v about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation.
Find the motional emf $\mathcal{E}$ induced between the ends of the bar.

Evaluate the magnitude of the emf induced in a segment of the bar of length dr having a velocity $\boldsymbol{v}$

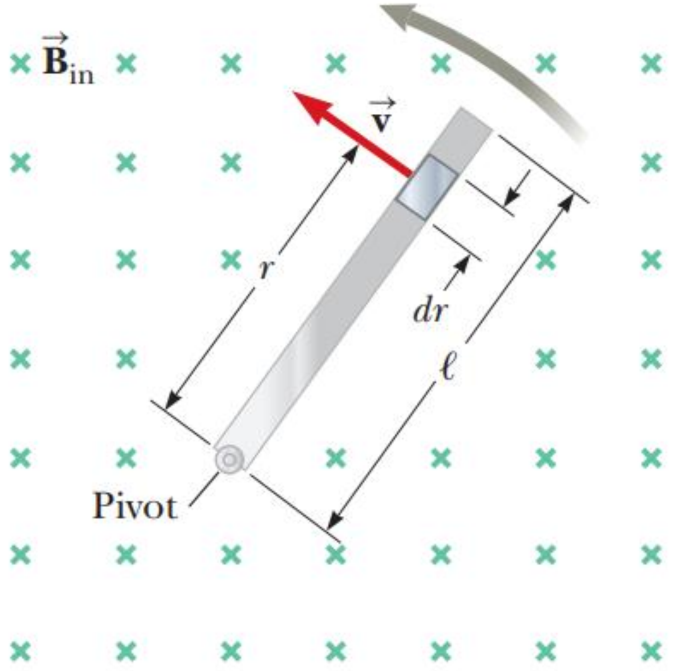
$$d\mathcal{E} = Bv\,dr$$

Find the total emf between the ends of the bar by adding the emfs induced across all segments:

$$\mathcal{E} = \int Bv\,dr$$

The tangential speed v of an element is related to the angular speed $\boldsymbol{v}$ through the relationship $v = \omega r$; use that fact and integrate:

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^\ell r\,dr = \frac{1}{2}B\omega\ell^2$$



We can increase $\mathcal{E}$ by increasing B, l or v. For the rotating rod, however, there is an advantage to increasing the length of the rod to raise the emf because l is squared. Doubling the length gives four times the emf, whereas doubling the angular speed only doubles the emf.

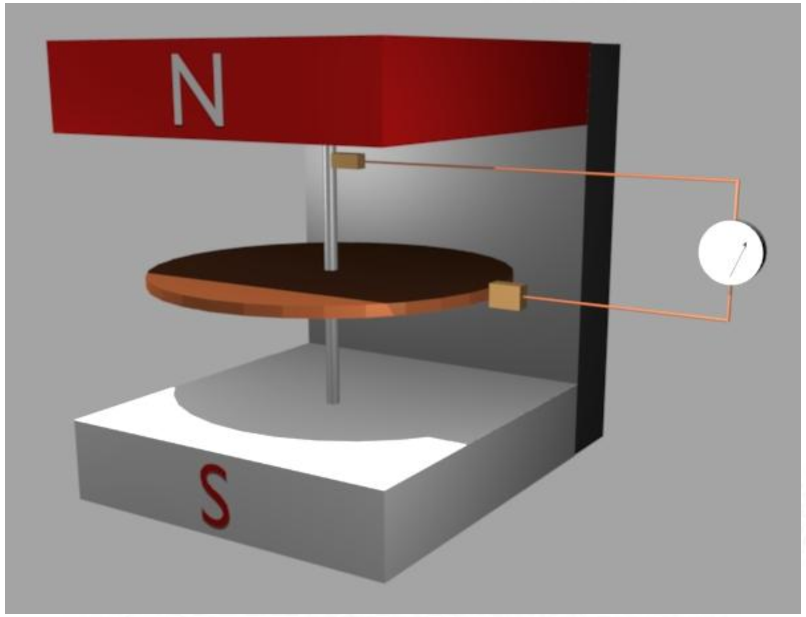
DC Electric Generator (Faraday disk)

Disc is conductive, spinning counterclockwise with ω

B is directed down.

Consider a portion of disk at distance r from the center, the Lorentz force on free electrons is:

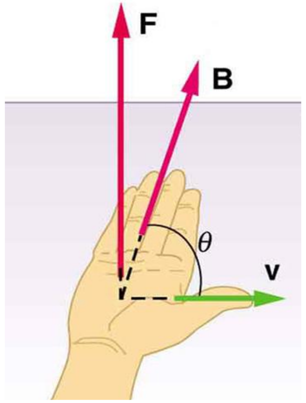
$$\vec{F} = q \cdot \vec{\mathbf{v}} \times \mathbf{B} \qquad v = \omega r$$

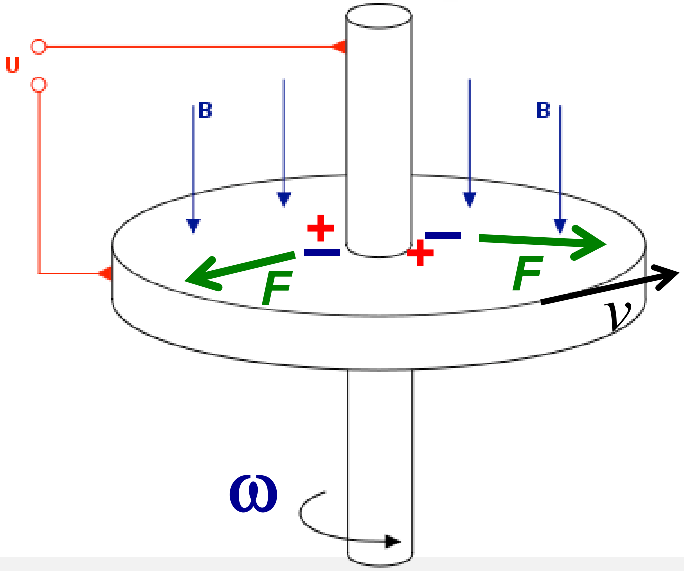


=> charge separation =>
Potential difference (*EMF*)

$$emf = \Delta V = Fl/e$$

l is the lengh across which we accumulated the charges, in this case $l = R$, the radius of the metallic disc.





$$I_{max} = \frac{\Delta V}{r_{disk}}$$

r_{disk} is the disk resistance, which can be very low if made our of thick metal

Capable to generate very high currents: 10^6 A !

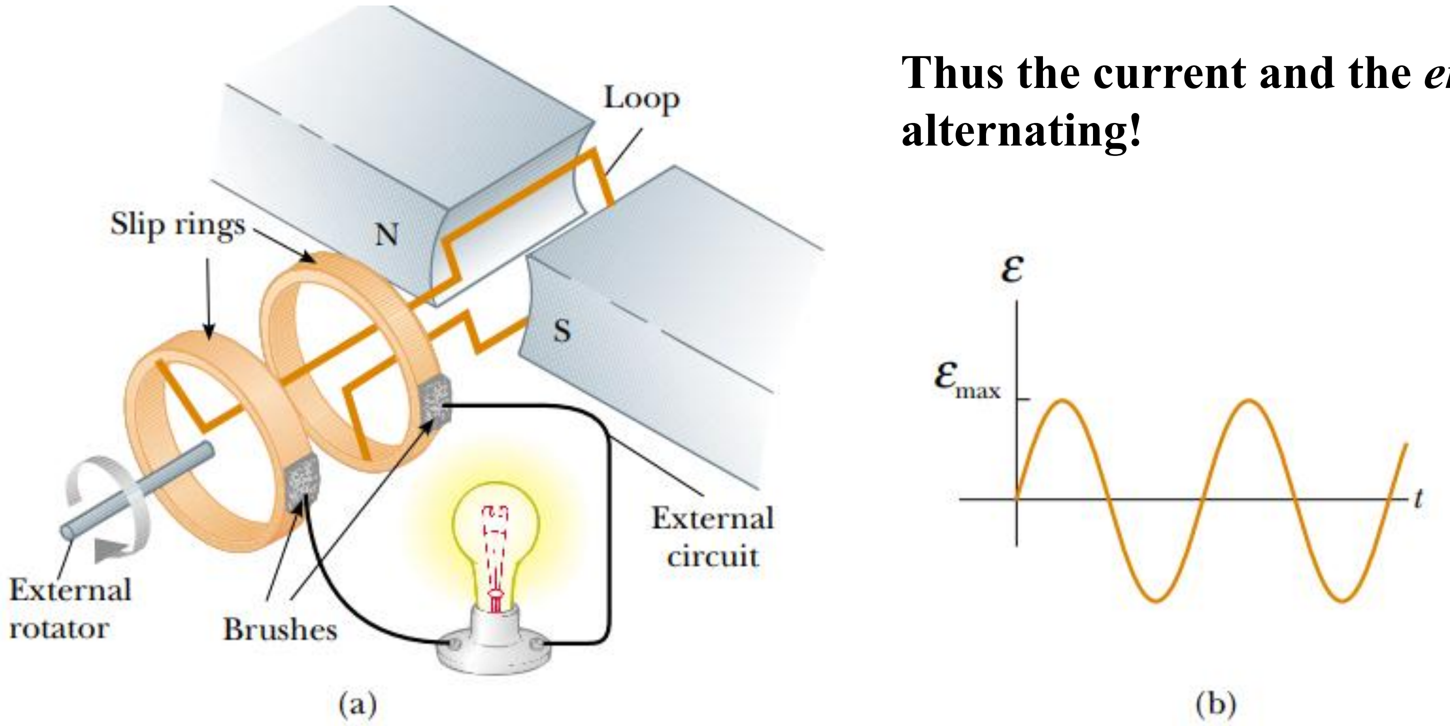
But low *emf*

AC Electric Generators

An alternating currents (ac) are generated by an electric generator or dynamo, one of the most important practical results of Faraday's great discovery. A generator transforms mechanical energy into electric energy, just the opposite of what a motor does.

A generator consists of many loops of wire (only one is shown) wound on an armature that can rotate in a magnetic field. The axle is turned by some mechanical means (falling water, steam turbine, car motor belt), and an ***emf*** is induced in the rotating coil. An electric current is thus the output of a generator.

Thus the current and the *emf* produced is alternating!



Active Figure 31.21 (a) Schematic diagram of an AC generator. An emf is induced in a loop that rotates in a magnetic field. (b) The alternating emf induced in the loop plotted as a function of time.

AC Electric Generators

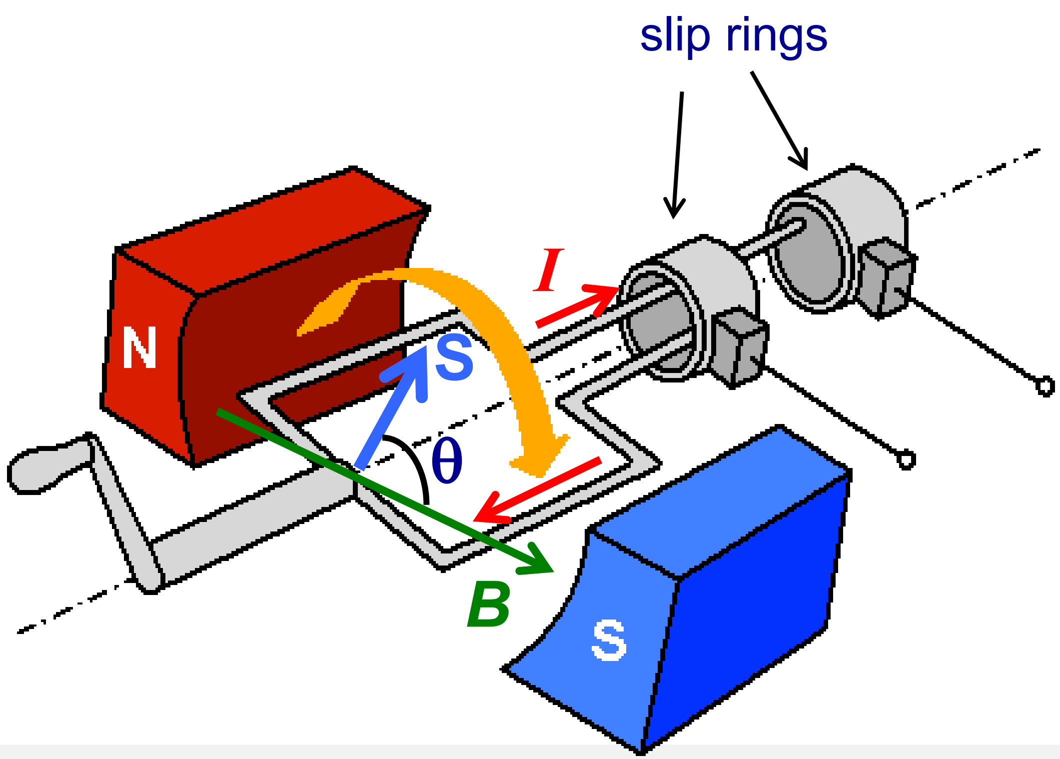
Much more efficient generator, respect to a DC one, involves a loop, (or a coil of many loops), rotating in a magnetic field generated between the poles of a magnet.

The brushes are in constant electrical contact with the *slip rings*, which are always connected to the same sides of the loop:

If the flux through the loop increases:

Lenz law

Current is induced into the loop with a direction is such as to reduce the magnetic field in the loop.



AC Electric Generators

In an electric generator, the axle is rotated by an external force (e.g., falling water or steam).

When the frame is horizontal ($\theta=90°$), the flux is 0.

It then increases and at $\theta=0°$ is max, flux is *BS*.

It then reduces, etc.: Φ = – + – + – +

By Lenz' law: **ε = + – + – + –** **Alternating current (AC)**

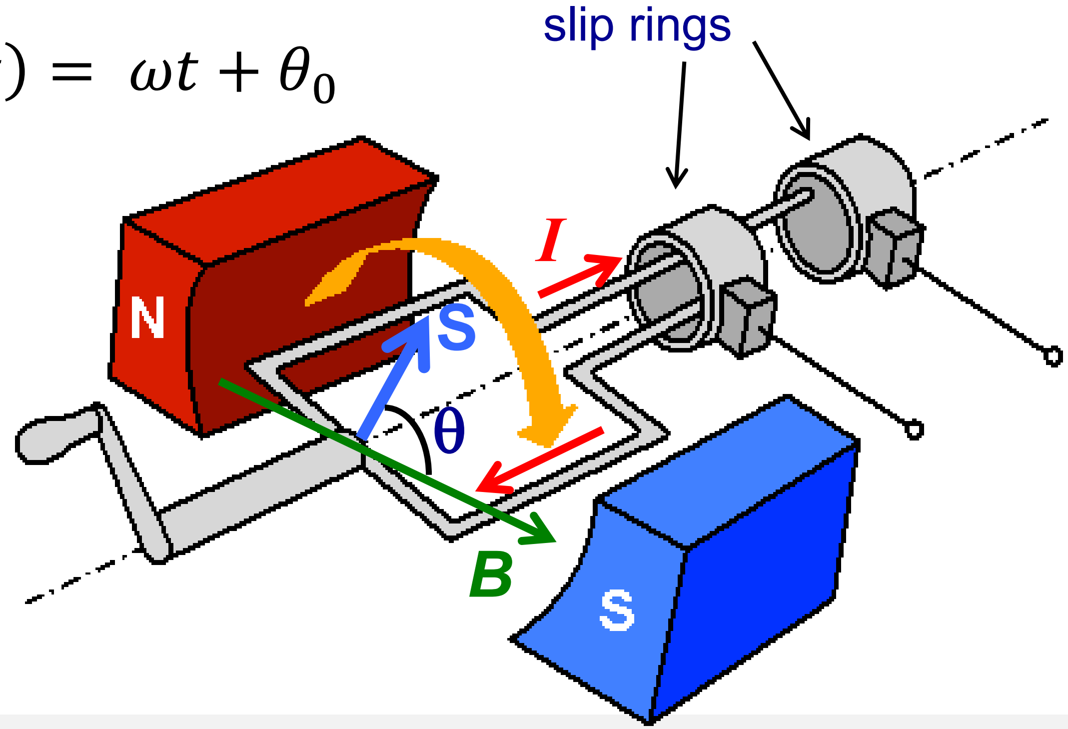
If rotation is constant: $\theta(t) = \omega t + \theta_0$

$$\varepsilon = -\frac{d\Phi}{dt} \; ; \quad \Phi = BScos\theta(t)$$

$$\varepsilon = -BS\frac{d}{dt}[\cos(\omega t + \theta_0)]$$

$$\varepsilon = -BS\frac{d\,cosy}{dy}\frac{d(\omega t + \theta_0)}{dt}$$

$$\varepsilon = BS\omega\sin(\omega t + \theta_0)$$



AC Electric Generators

$$\varepsilon = BS\omega \sin(\omega t + \theta_0)$$

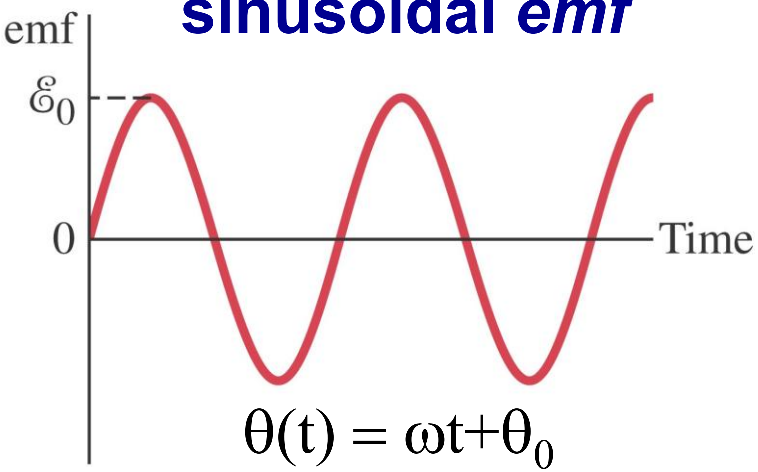
For the case of *N* loops:

$$\varepsilon = NBS\omega \sin(\omega t + \theta_0)$$

$$\varepsilon = \varepsilon_0 \sin(\omega t + \theta_0) \qquad \varepsilon_0 = NBS\omega$$

$$V = V_0 \sin(2\pi f t + \theta_0)$$

sinusoidal *emf*



$$\theta(t) = \omega t + \theta_0$$

Power by alternating current (AC) on a resistor:

Instantaneous power

$$P(t) = V(t)I(t) = \frac{V^2}{R} = \frac{V_0^2\, sin^2(2\pi f t)}{R}$$

Averaged Power over a period

$$\langle P \rangle = \frac{1}{T}\int_0^{T=1/f} P(t)dt = \frac{V_0^2}{RT}\int_0^T sin^2(2\pi f t)\, dt \qquad \boxed{\langle P \rangle = \frac{V_0^2}{2R}}$$

$$\sin^2 x + \cos^2 x = 1; \quad \langle \sin^2 x + \cos^2 x \rangle = \langle \sin^2 x \rangle + \langle \cos^2 x \rangle = 1$$

$$\langle \sin^2 x \rangle = \langle \cos^2 x \rangle = 1/2;$$

Induction by AC: Induction stove

An induction stove contains a coil of copper wire underneath the ceramic plate, the "burner" (a burner that never gets hot). When a cooking pot is placed on top of it, an alternating electric current is passed through it. The resulting oscillating magnetic field induces a magnetic flux, producing an eddy current in the ferrous pot.

Why will it heat a metal pan but not a glass container?

The eddy current flowing through the resistance of the pot heats it.

$$\varepsilon = -\frac{d\Phi_B}{dt} \approx -A\frac{d}{dt}B = -A\mu n\frac{d}{dt}\,I = -A\mu n\frac{d}{dt}\,(I_0 \cos\omega t)$$

assuming for a solenoid
$\boldsymbol{B} = \boldsymbol{\mu n I}$,
with n is the number of loops per unit length

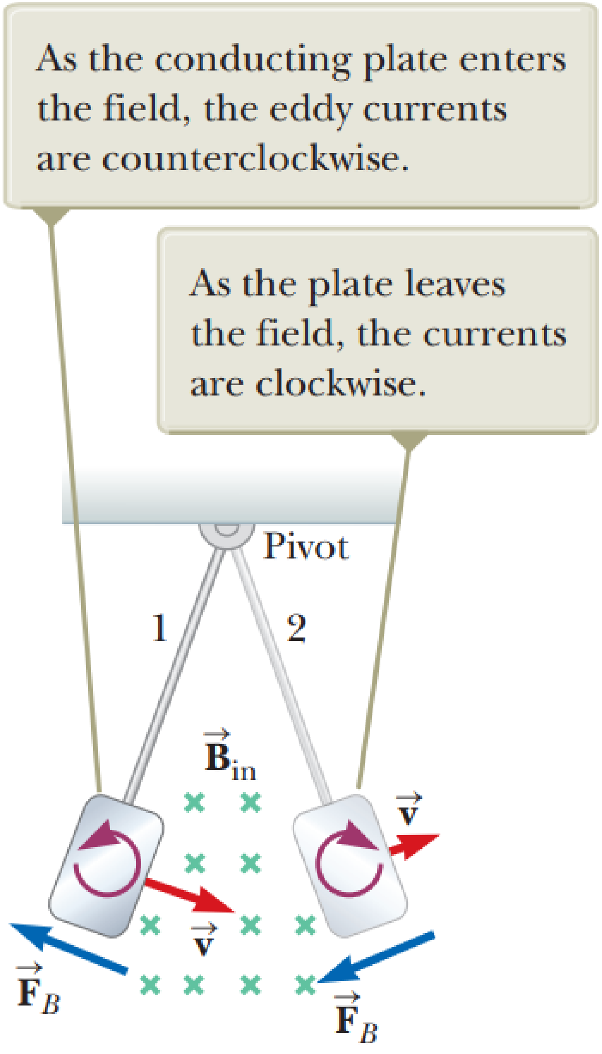
$$V \approx A\mu n I_0 \omega \; sin\,\omega t \qquad \langle P\rangle_t = \frac{\langle V^2\rangle_t}{R} = \frac{(A\mu n I_0\omega)^2}{R}\langle(\sin\omega t)^2\rangle_t$$

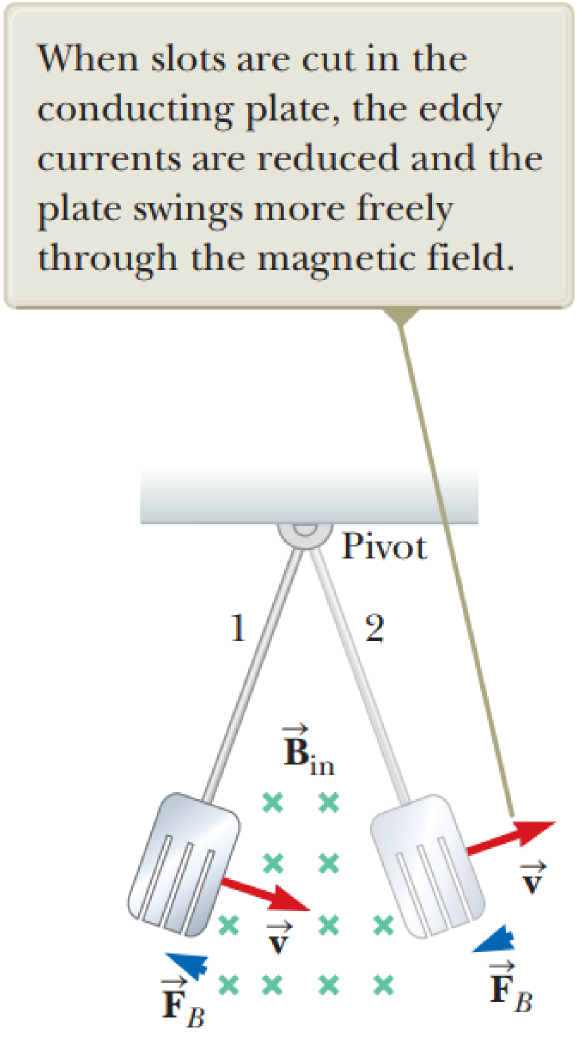
$$\langle P\rangle_t = \frac{(A\mu n I_0\omega)^2}{2R} \qquad \mu_{glass} << \mu_0 << \mu_{metal}$$

Important: For induction to work, your cookware must be made of a magnetic metal, such as cast iron or some stainless steels.

Eddy Currents

Circulating currents, called *eddy currents*, are induced in bulk pieces of metal moving through a magnetic field.

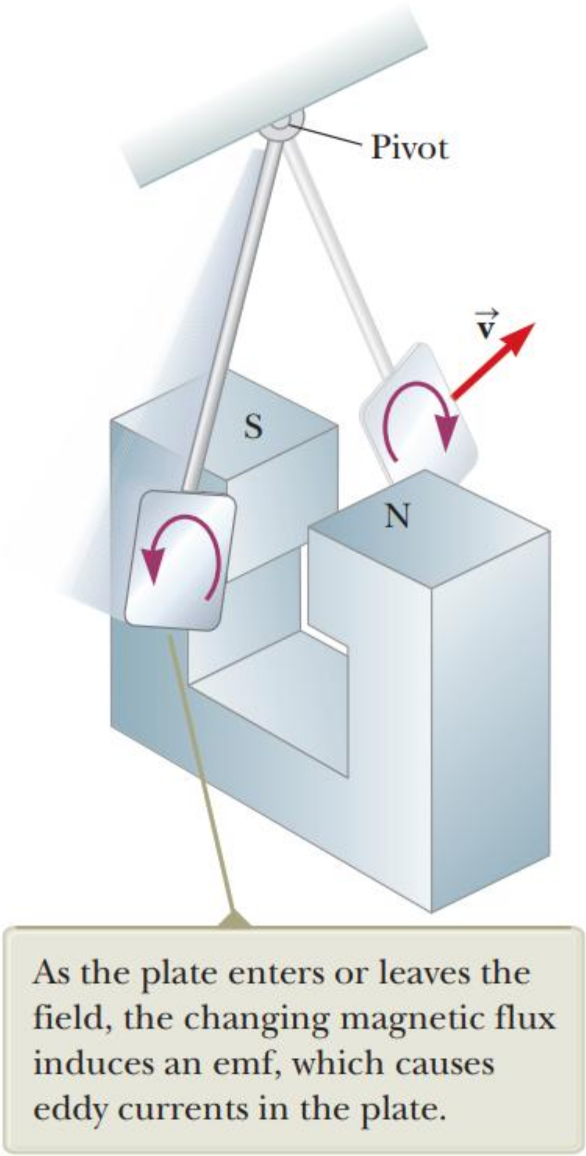




by Lenz's law, the induced current must provide its own magnetic field out of the page

DEMO

https://auditoires-physique.epfl.ch/experiment/595/freinage-dans-champ-magnetique-gros-aimant



Summary

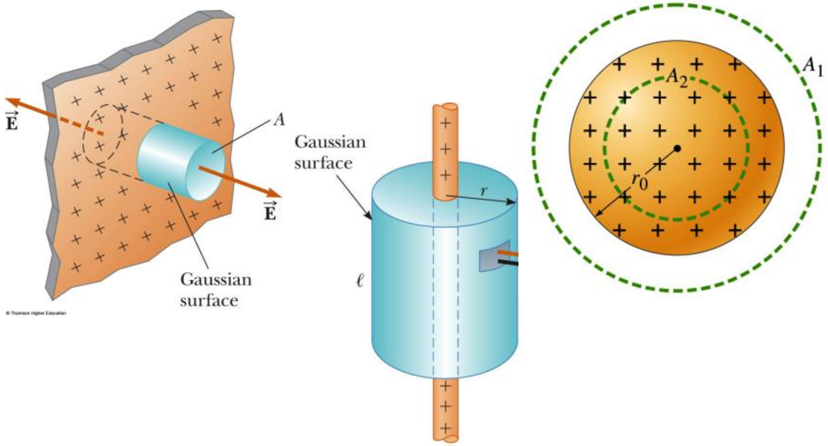
Field **E** created by a distribution of **static charges**

For problems with "symmetries" also used:

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\varepsilon_0} \oint_V \rho dV$$

(Gauss' law)

Surface integral



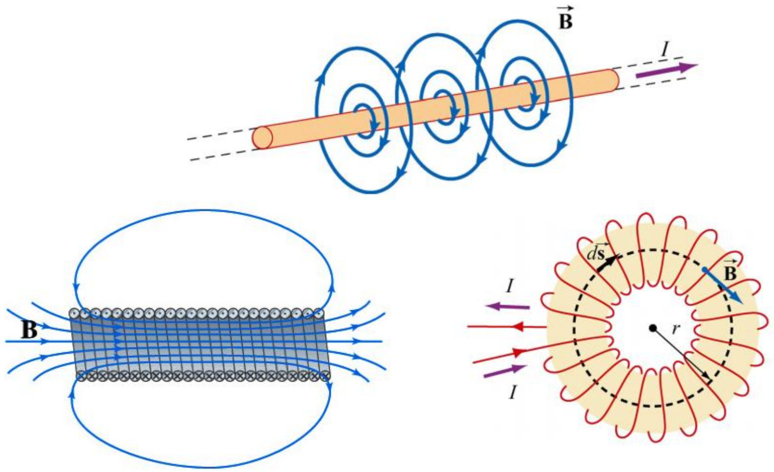
Field **B** created by a distribution of **stationary currents**

For problems with "symmetries" we also use:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{s}$$

(Ampere's Law)

Line integral

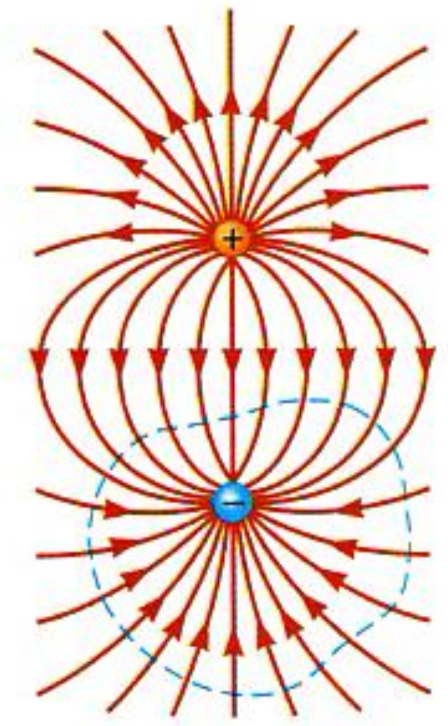


Summary

The flow of the **electric** field

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \int_V \frac{\rho}{\varepsilon_0} dV$$

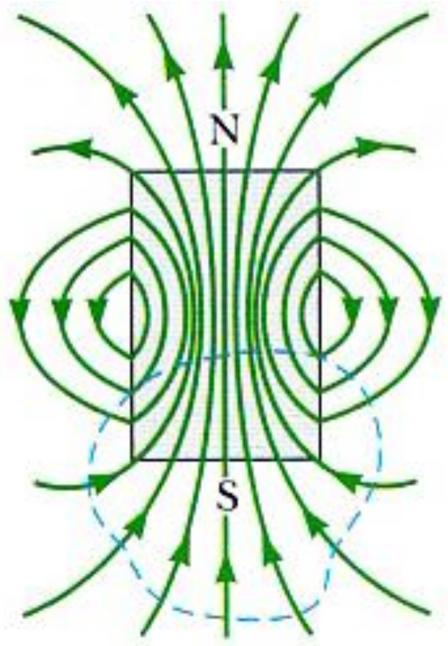
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$



The flux of the **magnetic** field

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

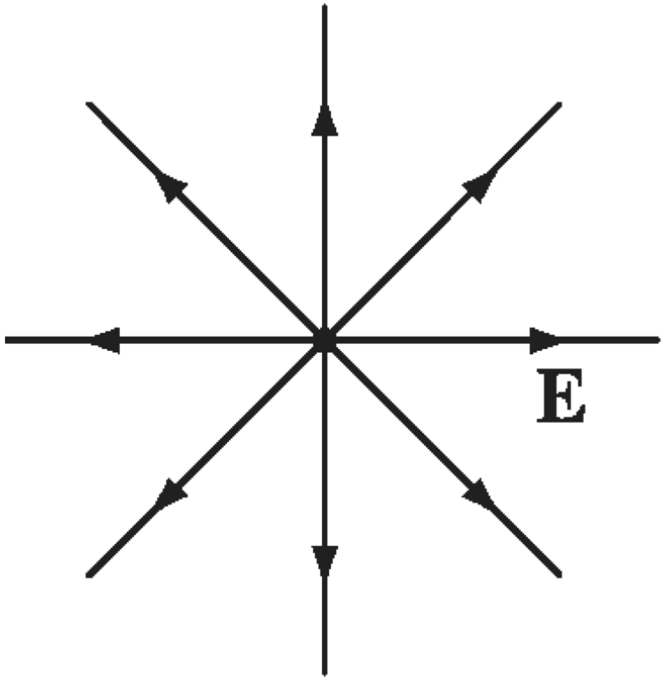
$$\nabla \cdot \mathbf{B} = 0$$



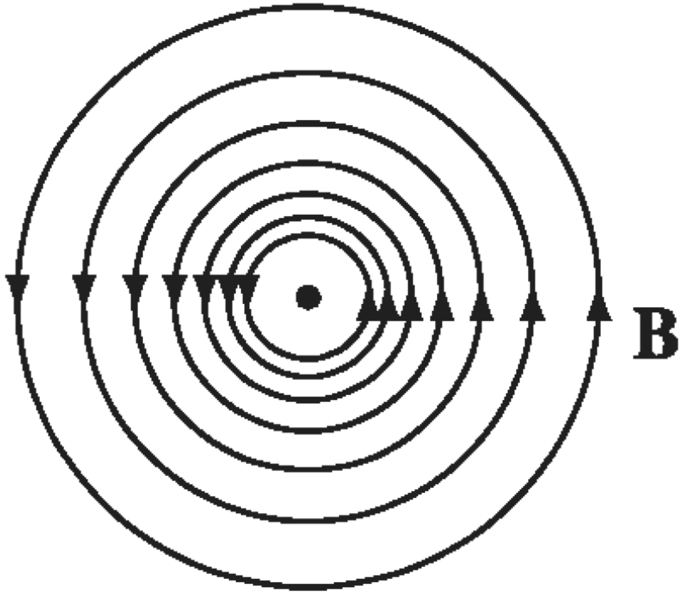
Interpretation:
Absence of "magnetic charges" (monopoles)
Field B lines are always closed (no springs or wells)

Summary

Electric field
of a point charge



Magnetic field
of a long wire



Energy density
electrostatic

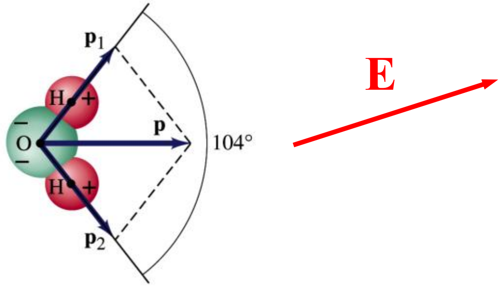
$$u_E = \frac{1}{2}\mathbf{E}\cdot\mathbf{E}$$

Energy density
magnetostatic

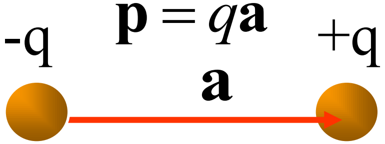
$$u_B = \frac{1}{2}\mathbf{B}\cdot\mathbf{B}$$

Summary

electric dipole
in a electric field

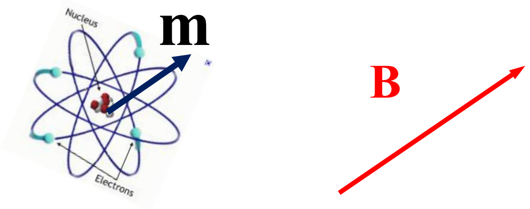


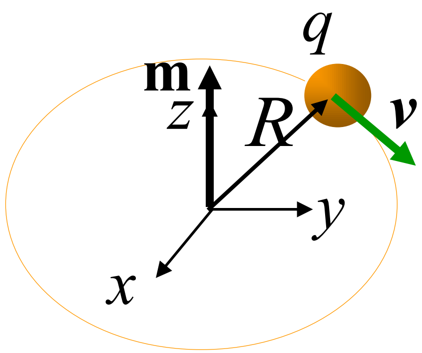
$\mathbf{p} = q\mathbf{a}$

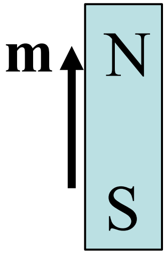


$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$
$$U_E = -\mathbf{p} \cdot \mathbf{E}$$

Magnetic dipole
in a magnetic field







$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$
$$U_B = -\mathbf{m} \cdot \mathbf{B}$$